Draw the Golden Ratio, or Golden Proportion, or Divine Proportion

The Golden Number(s) in Geometrical Representation

By Ion Saliu

I. The Mathematics of the Golden Ratio

Let's take a segment and divide it into two parts, **a** and **b**. There are two conditions that lead to the Golden Ratio or Golden Proportion:

1) **a** is greater than **b** ($\mathbf{a > b}$); AND
2) $\frac{\mathbf{a}}{\mathbf{a+b}} = \frac{\mathbf{b}}{\mathbf{a}}$

For the sake of simplification, we can make $\mathbf{a + b = 1}$ (i.e. any unit of measure is valid; we don't have to take a segment of integers, such as 8 inches or 10 centimeters).

$a + b = 1$ and $b = 1 - a$; therefore:

$\frac{a}{a+1-a} = \frac{1-a}{a}$ or:

$a = \frac{1-a}{a}$ or the famous equation:

$\mathbf{x^2 + x - 1 = 0}$, where:

$a = b = c = 1$, therefore:

$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ (the ***quadratic formula***).

$$\boldsymbol{\Phi} = \frac{-1 \pm \sqrt{5}}{2} = 0.618033989...$$

Or: $\boldsymbol{\Phi} = 1.618033989...$ (false golden number, as the root is negative)

The common belief is that there are **two** golden numbers. We can express them as:

$\Phi 1 = \frac{\sqrt{5}-1}{2}$ (0.618033989…) and

$\Phi 2 = \frac{\sqrt{5}+1}{2}$ (1.618033989…)

In truth, the Ancient Greeks dealt with one and only one golden number. They did not work with negative numbers. They didn't know about the quadratic formula. Moreover, the Pythagoreans were adverse to the irrational numbers (such as $\sqrt{5}$, the core of the golden ratio).

The Ancient Greeks dealt with the golden proportion (they considered it transcendental or divine) ***geometrically*** only. The secretive mathematicians and artists drew segments and arcs that divided into this beautiful ratio. The sculpture, painting, architecture of Ancient Greece are founded on the divine proportion.

Thus, we can make the strong statement that the true and only ***golden number*** is:

$$\boldsymbol{\Phi} = \frac{\sqrt{5}-1}{2} \ (0.618033989...)$$

$$\frac{1}{\Phi} = \Phi + 1$$

$$\frac{1}{\Phi} - 1 = \Phi$$

It is so because the entire segment is considered one unit. An entity cannot be divided into parts larger than itself — it is an absurdity!

Let's demonstrate that $\Phi + 1 = \frac{1}{\Phi}$

$\Phi = \frac{\sqrt{5}-1}{2}$ and $\frac{1}{\Phi} = \frac{2}{(\sqrt{5}-1)}$

$\Phi + 1 = \frac{\sqrt{5}-1}{2} + \frac{2}{2} = \frac{\sqrt{5}+1}{2}$

Is $\frac{2}{(\sqrt{5}-1)} = \frac{\sqrt{5}+1}{2}$?

$2 * 2 = (\sqrt{5} - 1) * (\sqrt{5} + 1) = 5 + \sqrt{5} - \sqrt{5} - 1 = 4$

QED

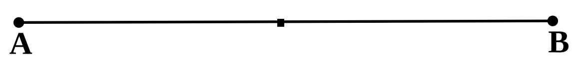
II. Bisect the Segment (Divide it into Two Equal Parts)

Let's draw the ***Golden Proportion*** the same way the Ancient Greeks did.

It is very important to determine the **exact** midpoint of the segment. Measuring with a ruler may never be exact, especially when the length of the segment is not compatible with integers. That's why the Ancients chanted *"I am the mean and the extreme"*.

I will not draw here, as I am not proficient with drawing software. But following the instructions makes drawing very easy. We need a **compass** (to draw circles or arcs) and s**et squares** or a **straightedge** (to draw perpendicular lines).

Let's take a segment **AB**.



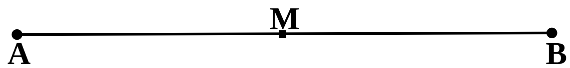
Place the compass on one end of the segment (e.g. point **A**). Set the compass width to three quarters the line length. The actual width does not matter, as long as it is longer than the half of the segment, but no longer than its dimension or length. Without changing the compass width, draw an arc on each side of the line (segment).

Also without changing the compass width, place the compass point on point **B** of the segment. Draw an arc on each side of the line so that the arcs cross the first two.

Using set squares (or a straightedge), draw a line between the points where the arcs intersect.

This line is perpendicular to the segment **AB** and bisects it (cuts it at the exact midpoint of the segment). Mark this point **M**. You can make it easier to draw if you erase the perpendicular line that created the midpoint **M**.

III. The Geometry: Draw the Golden Ratio



1) Place the compass on one end of the segment (I prefer the right point, **B**). Set the compass width to half the length (to point **M**). Without changing the compass width, draw an arc above the line (segment).

2) Using set squares (or a straightedge), draw a perpendicular line above point **B** until it intersects the arc. Mark the intersection as point **C.**

3) Using a straightedge, draw a line between point **A** and point **C** (the hypotenuse **AC**). The resulting figure is the right triangle **ABC**. (For clarity, you might want to erase the arc.)

In this case, we have the following dimensions:

$$\mathbf{AB} = 1$$

$$\mathbf{BC} = \frac{1}{2}$$

We apply ***Pythagoras' Theorem*** :

$$\mathbf{AB^2} + \boldsymbol{BC^2} = \mathbf{AC^2}$$

$$\mathbf{AC}^2 = 1 + \frac{1}{4} = \frac{5}{4}$$

The hypotenuse is:

$$\mathbf{AC} = \frac{\sqrt{5}}{2}$$

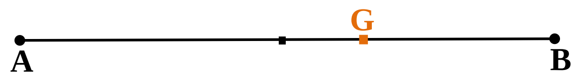
I call $\sqrt{\mathbf{5}}$ the ***Heart of the Divine Proportion***.

4) Place the compass on point **C**. Set the compass width to half the length of segment **AB** (point **B** to point **M**). Without changing the compass width, draw an arc to intersect the hypotenuse **AC).** Mark the intersection as point **D**.

5) We already created a segment equal to Φ (0.618033989…) point **A** to point **D.** Here's the math:

$$\mathbf{AD} = \frac{\sqrt{5}}{2} - \frac{1}{2} = \frac{\sqrt{5}-1}{2}$$

6) Place the compass on point **A.** Set the compass width to point **D** (the distance **AD** represents the golden number $\Phi = 0.618033989\ldots$) Without changing the compass width, draw an arc to intersect the original segment **AB.** Mark the intersection as point **G** (as in GOLDEN Ratio).



7) The distance **AG** is equal to the golden number Φ (0.618033989...) The point **G** divides the segment **AB** into two segments, **AG** = **a** and **GB** = **b**. The division satisfies the golden proportion relation:

$$\frac{\mathbf{a}}{\mathbf{a+b}} = \frac{\mathbf{b}}{\mathbf{a}}$$

$$\frac{\mathbf{a}}{\mathbf{a+b}} = \frac{\sqrt{5}-1}{2} / 1 = \frac{\sqrt{5}-1}{2}$$

How about the second part of the equation, $\frac{\mathbf{b}}{\mathbf{a}}$? We can see that segment **b** represents the difference between the length of segment AB $(\mathbf{a} + \mathbf{b} = \mathbf{1})$ and segment **a** (equal to Φ or $\frac{\sqrt{5}-1}{2}$).

$$\mathbf{b} = 1 - \frac{\sqrt{5}-1}{2} = \frac{2-(\sqrt{5}-1)}{(\sqrt{5}-1)} = \frac{3-\sqrt{5}}{2}$$

Therefore:

$$\frac{b}{a} = \frac{3-\sqrt{5}}{2} * \frac{2}{(\sqrt{5}-1)} = \frac{2-(\sqrt{5}-1)}{(\sqrt{5}-1)} = \frac{2}{(\sqrt{5}-1)} - 1 = \mathbf{0.618033989\ldots}$$

We know that $\frac{2}{(\sqrt{5}-1)} = \frac{1}{\Phi} = \mathbf{1.618033989\ldots}$

Evidently:

$$\frac{\mathbf{b}}{\mathbf{a}} = \frac{\sqrt{5}-1}{2} \text{ or}$$

$$\frac{\mathbf{b}}{\mathbf{a}} = \frac{\mathbf{1}}{\boldsymbol{\Phi}} - 1 = \boldsymbol{\Phi}$$

That's how the Ancient Greeks (the initiated ones) constructed the Golden Ratio. Of course, we can reverse the initial equation (it becomes **large** over **small**):

$$\frac{\mathbf{a+b}}{\mathbf{a}} = \frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{1}}{\boldsymbol{\Phi}} = 1.618033989...$$

Remember, there is NO **+1.618033989...** when we apply the quadratic formula. There is only one positive root (**0.618033989...);** there is also one negative root: **–1.618033989...** The Classical Greeks did not recognize negative numbers — and they were right. The Reality consists of positive values only. Additionally, when we divide we take a value and mark it into **smaller** parts.

In conclusion, we divided the segment **AB** into two segments that represent the ***Golden Ratio***, or ***Golden Proportion***, or ***Divine Proportion***.

I believe this is the best method to create every shape of the Golden Ratio. The segment can create the largest variety of shapes. The ratio as created here can be used to create the ***golden spirals*** as well.

Back PI Day, Divine Proportion, Golden Ratio, Golden Number, Fibonacci, PHI